INTEGRATION OF IRRATIONAL FUNCTION

(IV) $\int \frac{dx}{\sqrt{x^2+a^2}}$

Put $x = a\sinh\theta \quad \therefore dx = a\cosh\theta$

$\therefore I = \int \frac{a\cosh\theta\, d\theta}{\sqrt{a^2\sinh^2\theta + a^2}} = \int \frac{a\cosh\theta\, d\theta}{a\cosh\theta}$

$\Rightarrow \int d\theta$

$\Rightarrow \theta = \sinh^{-1}\frac{x}{a}$

$= \log\left(\frac{x+\sqrt{x^2+a^2}}{a}\right) + C$

$= \log(x+\sqrt{x^2+a^2}) - \log a + C$

C

$= \log(x+\sqrt{x^2+a^2})$

(v) $\int \frac{dx}{\sqrt{x^2-a^2}}$ $\quad x > a$

Put $x = a\cosh\theta$ $\quad \therefore dx = a\sinh\theta\, d\theta$

$\therefore I = \int \frac{a\sinh\theta\, d\theta}{\sqrt{a^2\cosh^2\theta - a^2}} =$

$\Rightarrow \int \frac{a\sinh\theta}{a\sqrt{\cosh^2\theta - 1}}\, d\theta$

$\Rightarrow \int \frac{a\sinh\theta}{a\sinh\theta}\, d\theta.$

$\Rightarrow \int d\theta = \theta = \cosh^{-1}\frac{x}{a}.$

$= \frac{\log x + \sqrt{x^2-a^2}}{a}.$